Exam. Code : 211001
Subject Code : 4847

M.Sc. Mathematics 1st Semester

COMPLEX ANALYSIS

Paper—MATH-552

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Attempt **FIVE** questions in all, selecting at least **ONE** question from each section. The **fifth** question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Show that a function $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain iff v is a harmonic conjugate of u.

   (b) Prove that :

$$\frac{dw}{dz} = e^{-i\theta}\frac{\partial w}{\partial r} = -\frac{1}{r}e^{-i\theta}\frac{\partial w}{\partial \theta}.$$

2. State and prove Cauchy's integral theorem.

SECTION—B

3. (a) State and prove Poisson's integral formula.

   (b) Show that each analytic function with non-vanishing derivative is conformal in each region.

4. (a) Define cross-ratio. Show that cross-ratio are invariant under a bilinear transformation.

   (b) Show that the power series $\sum_{n=q}^{\infty} z^{3^n}$ cannot be continued analytically beyond the circle $|z| = 1$.

SECTION—C

5. (a) Show that $\exp\left[\frac{c}{2}\left(z-\frac{1}{z}\right)\right]=\sum_{n=-\infty}^{\infty} a_n z^n$ where

$$a_n = \frac{1}{2\pi}\int_0^{2\pi} \cos(n\theta - c\sin\theta)\,d\theta .$$

(b) State and prove Schwarz's lemma.

6. (a) State and prove Argument principle.

(b) Apply Rouche's theorem to find the number of zeros of the polynomial $2z^4 - 2z^3 + z^2 + 11$ inside the circle $|z| = 1$.

SECTION—D

7. (a) Find the residues at the poles of the function

$$\frac{z^4}{(c^2+z^2)^4}.$$

(b) Evaluate $\int_0^{\pi} \frac{a\,d\phi}{a^2+\cos^2\phi}$, where a is positive.

8. (a) State and prove Jordan's lemma.

(b) Apply the calculus of residues to prove that :

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^3} = \frac{3\pi}{8}.$$